

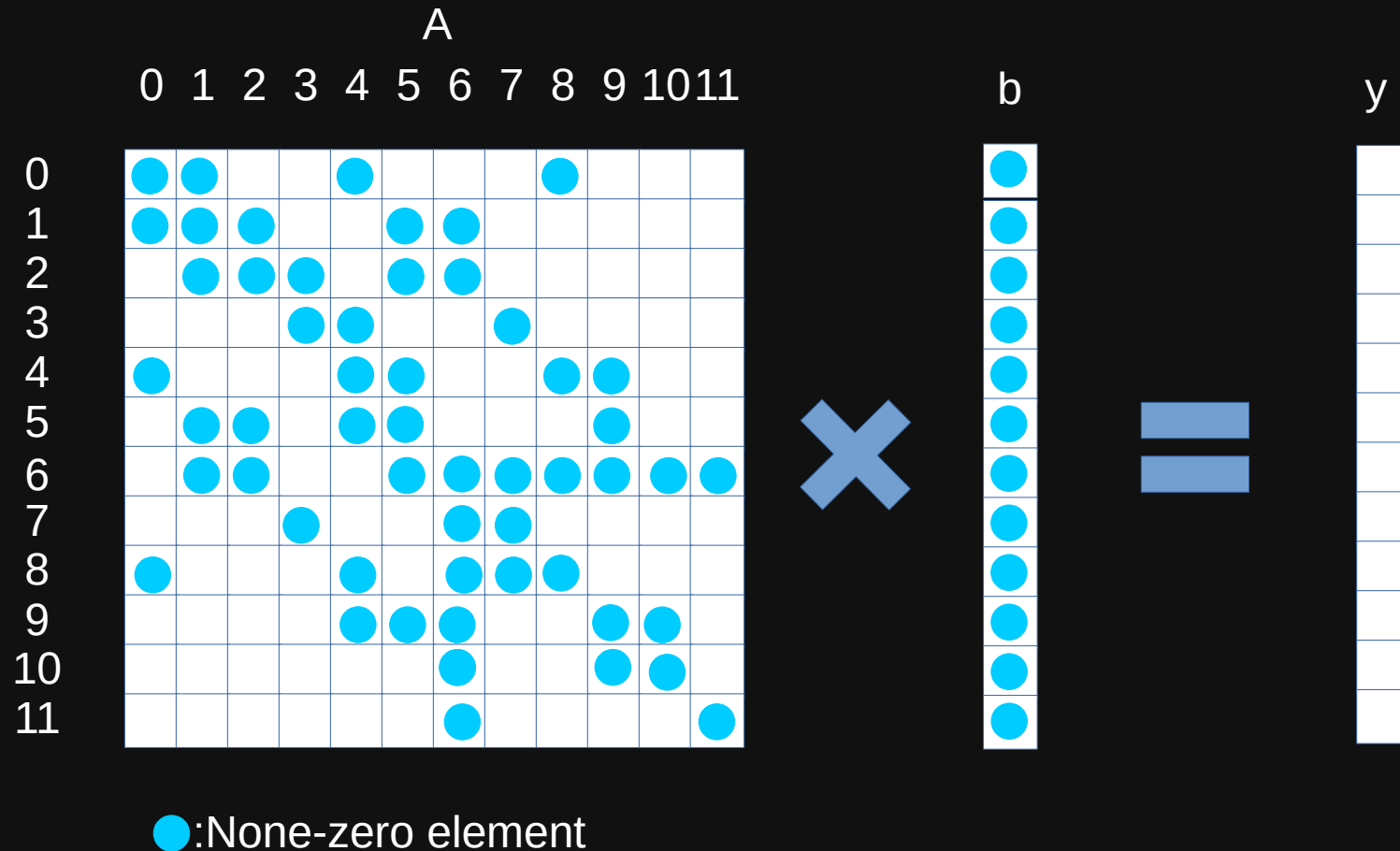
Parallel Algorithm Design Case Study: Sparse Matrix Vector Multiplication

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A Slightly More Complex Parallel Algorithm

- So far the examples we have seen are all very simple and naive
- Sparse matrix operations are popular in parallel algorithm studies
 - A sparse matrix is a matrix in which most of the elements are zero
 - Sparse matrices are very common in nature
 - Sparse matrices have many temporal and spatial optimization opportunities
- The case study here is still a very simple case of sparse matrix operations

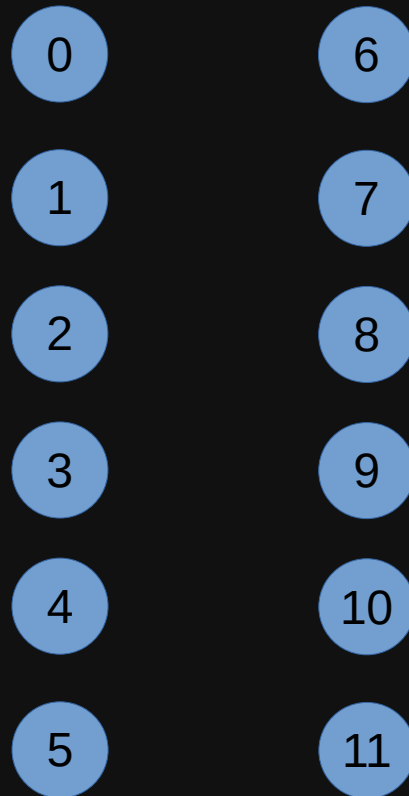
A Example of Sparse Matrix Vector Multiplication



A Example of Sparse Matrix Vector Multiplication – Task Decomposition

- Task partitioning: each task computes one element of y
 - Task0: $y[0] = A[0,0] * b[0] + A[0,1] * b[1] + A[0,4] * b[4] + A[0,8] * b[8]$
 - Task1: $y[1] = A[1,0] * b[0] + A[1,1] * b[1] + A[1,2] * b[2] + A[1,5] * b[5] + A[1,6] * b[6]$
 - Task2: $y[2] = A[2,1] * b[1] + A[2,2] * b[2] + A[2,3] * b[3] + A[1,5] * b[5] + A[1,6] * b[6]$
 - Task3: $y[3] = A[3,2] * b[2] + A[3,3] * b[3] + A[3,7] * b[7]$
 - Task4: $y[4] = A[4,0] * b[0] + A[4,4] * b[4] + A[4,5] * b[5] + A[4,8] * b[8] + A[4,9] * b[9]$
 - Task5: $y[5] = A[5,1] * b[1] + A[5,2] * b[2] + A[5,4] * b[4] + A[5,5] * b[5] + A[5,6] * b[6] + A[5,9] * b[9]$
 - Task6: $y[6] = A[6,1] * b[1] + A[6,2] * b[2] + A[6,5] * b[5] + A[6,6] * b[6] + A[6,7] * b[7] + A[6,8] * b[8] + A[6,9] * b[9] + A[6,10] * b[10] + A[6,11] * b[11]$
 - Task7: $y[7] = A[7,3] * b[3] + A[7,6] * b[6] + A[7,7] * b[7]$
 - Task8: $y[8] = A[8,0] * b[0] + A[8,4] * b[4] + A[8,6] * b[6] + A[8,8] * b[8]$
 - Task9: $y[9] = A[9,4] * b[4] + A[9,5] * b[5] + A[9,6] * b[6] + A[9,9] * b[9] + A[9,10] * b[10]$
 - Task10: $y[10] = A[10,6] * b[6] + A[10,8] * b[8] + A[10,9] * b[9]$
 - Task11: $y[11] = A[11,6] * b[6] + A[11,11] * b[11]$

A Example of Sparse Matrix Vector Multiplication – Task Dependency



Tasks are all
independent

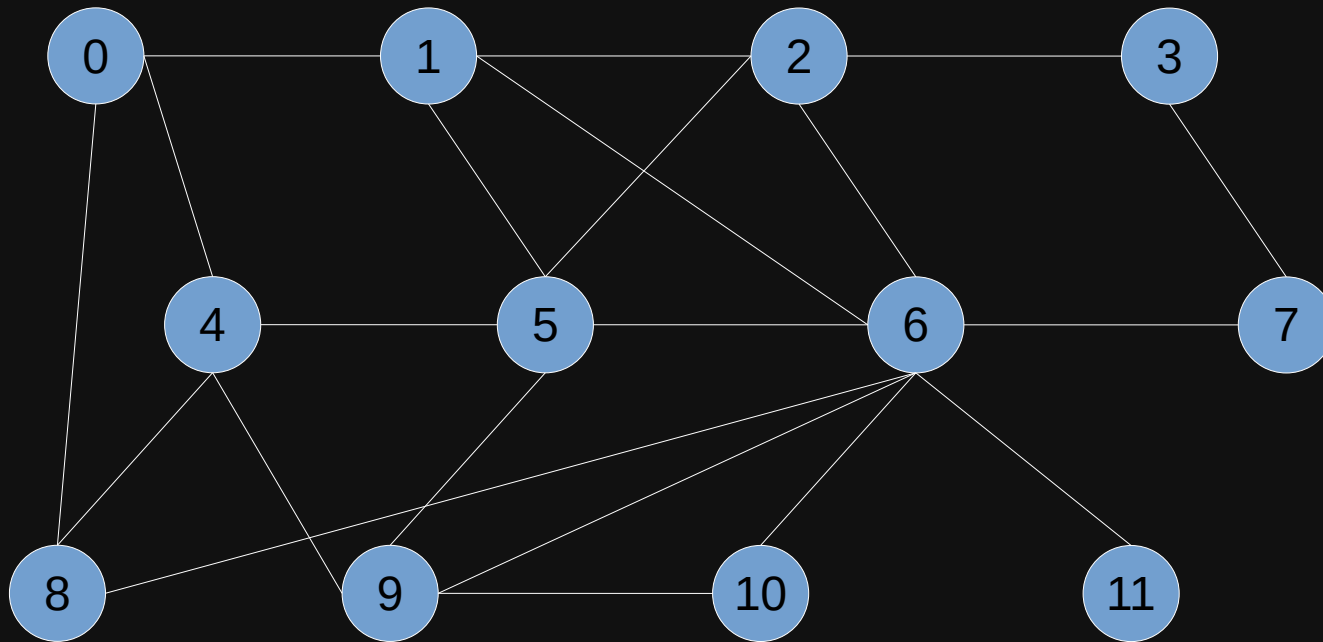
A Example of Sparse Matrix Vector Multiplication – Data Assignment

- For Task_i,
 - It is assigned with all data associated with row i of A , i.e., $A[i, *]$
 - It is assigned with $y[i]$
 - It is also assigned with $b[i]$
 - A common strategy in case b is a very large vector, and replicating b 's data at each task is impractical

A Example of Sparse Matrix Vector Multiplication – Communication

- Because $b[i]$ is assigned to Task_i , Task_i has to send $b[i]$ to any other tasks who uses $b[i]$
 - e.g., Task_0 has to send $b[0]$ to Task_1 , Task_4 , and Task_8
- Task interaction graph – illustrate the communication between tasks

A Example of Sparse Matrix Vector Multiplication – Task Interaction



A Example of Sparse Matrix Vector Multiplication – Task Mapping

