# Finite State Machine and Regular Expressions

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# Textbook Chapters

- This lecture corresponds to the chapters of 3.3 (regular expression), 3.6/3.7 (finite automata) and 3.5 (flex) of the Dragon book.

# Lexical Analysis

- Lexical analysis is the first step taken by a compiler
- Lexical analysis recognizes the tokens, such as identifiers, constants and keywords
- Lexical analysis is essentially pattern matching, which is achieved with regular expression and finite automata (state machines).

# Pattern Matching Basics

- Pattern matching is a well studied area
- Finite Automata constructed and used for all pattern matching tasks, e.g.,
  - String matching / processing
  - Lexical analysis
- Regular expressions (RE) are used to simplify pattern expression
- Lex or Flex are used to automatically convert patterns (RE) to finite automata to executable programs

# What We Will Learn

- Regular expression
- Finite Automata
  - Deterministic/Non-deterministic finite automata
  - Conversion from Non-deterministic to deterministic
  - Minimizing deterministic finite automata
- Flex
  - Write flex grammar to convert regular expression to a program that performs pattern matching

# Regular Expression

- Regular expression is an algebraic way to describe patterns/strings, more formally, languages.
- Regular expression contains:
  - Regular characters: means the character itself; most letters are regular characters
  - Special characters: special operations on regular characters
- ε is a special character represents empty string – a string without any character

# Regular Expression: Regular Characters

- Most letters, numbers and some punctuations are normal characters
- E.g., regular expression `abc` matches string “abc”, and only that string
- E.g., regular expression `x87z` matches string “x87z”, and only that string

# Regular Expression: Sub-expression and Concatenation

- Parentheses '(' and ')' mark an subexpression
  - e.g., regular expression `(abc)` matches string "abc", and only that string
  - e.g., regular expression `(x87z)` matches string "x87z", and only that string
- Subexpressions and regular characters can be concatenated into one regular expression
  - e.g., regular expression `(x87z)mu(abc)` matches string "x87zmuabc", and only that string

# Regular Expression: Special Characters

- *: matches zero or more of a sub-expression
  - e.g., `ab*` matching any string starts with an `a`, following by zero or more `b`'s, such as "a", "ab", "abb", "abbb", "abbbb" …
  - e.g., `(ab)*` matching any string that repeats "`ab`", including the empty string, such as ε, "ab", "abab", "ababab", "abababab" …

# Regular Expression: Special Characters cont'd

- +: matches one or more of a sub-expression
  - e.g., `ab+` matching any string starts with an `a`, following by one or more `b`'s, such as “ab”, “abb”, “abbb”, “abbbb” …
  - e.g., `(ab)+` matching any string that repeats “`ab`”, excluding the empty string, such as “ab”, “abab”, “ababab”, “abababab” …

# Regular Expression: Special Characters cont'd

- | : matches one or another
  - e.g., `ab|bc` matches “ab” or “bc”
  - e.g., `x(10|01)x` matches “x10x” or “x01x”
- . : matches one character
  - e.g., `a.b` matches any 3-character tring starts with `a` and ends with `b`, such as “acb”, “axb”, “a0b” …
  - e.g. `a.*b` matches any strings starts with `a` and ends with `b`, such as “axxxb”, “ab”, “a098xb” …

# Regular Expression: Special Characters cont'd

- [ and ]: matches a single character that is contained within the brackets.
  - e.g., `a[bc]d`, matches “abd” or “acd”
  - e.g., `x[0-9]y`, matches any string starts with `x`, ends with `y`, and has one digit in middle, i.e., “x0y”, “x1y”, “x2y”, … , “x9y”
  - e.g., `0[a-zA-Z]1`, matches any string starts with 0, ends with 1, and has letter in middle, suc as, “0x1”, “0q1”, “0L1”, …

# Regular Expression: Special Characters cont'd

- [^ and ] : matches any character that is not contained within the brackets
  - e.g., `xyz[^abc]`, matches any 4-character string starts with "`xyz`" and does not end with an `a`, `b` or `c`.
- { and }: specifies the number of occurrence of subexpression
  - e.g., `a{3,5}`, matches any string with 3 to 5 a's
  - e.g., `[0-9]{2,9}`, matches any string with 2 to 9 digits

# Regular Expression: Special Characters cont'd

- There are more special characters, defined by various standards. You can find them online.
- Sometimes, you need to put “\” before a special character for it to be recognized a special character
  - e.g., basic regular syntax of POSIX
- Sometimes, you need to put “\” before a special character for it to recognized a regular character
  - e.g., extended regular syntax of POSIX

# Some Regular Expression Examples

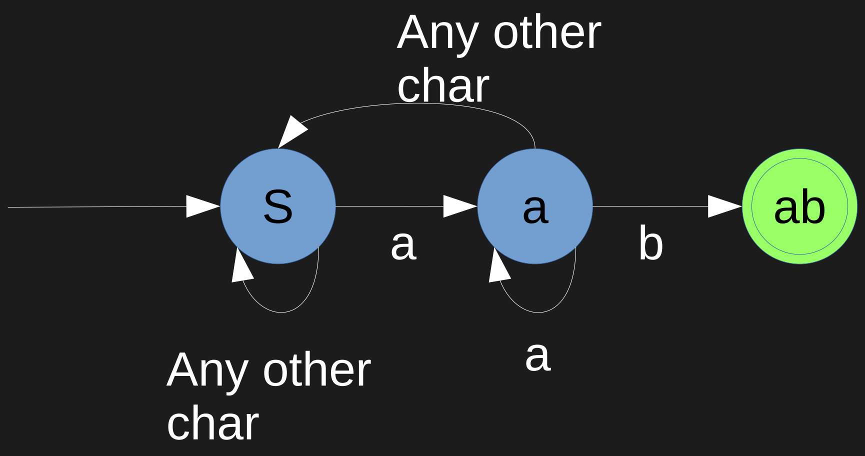
- A phone number;
  - [0-9]{3,3}\-[0-9]{3,3}\-[0-9]{4,4}
- An email address with only lower case characters, numbers, dot and @
  - [a-z][a-z0-9]*@[a-z0-9\.]*[a-z]

# Finite-state Automata

- Finite-state Automata is a simple idealized machine used to recognize patterns within input strings
- Non-deterministic Finite Automata (NFA):
  - Used to convert regular expressions into finite-state automata
- Deterministic Finite Automata (DFA):
  - Converted from NFA for better implementation of pattern matching
  - NFA and DFA are equivalent in pattern matching
- Constructing DFA is the standard approach for arbitrary pattern matching or substring matching

# A Finite Automaton Example

- This automaton matches any string with a substring "ab"
  - "S" is the start state
  - "ab" is the acceptance state (a match found)
  - "rej" is the rejection state (no match found)
  - An arrow represents a state change based on input character

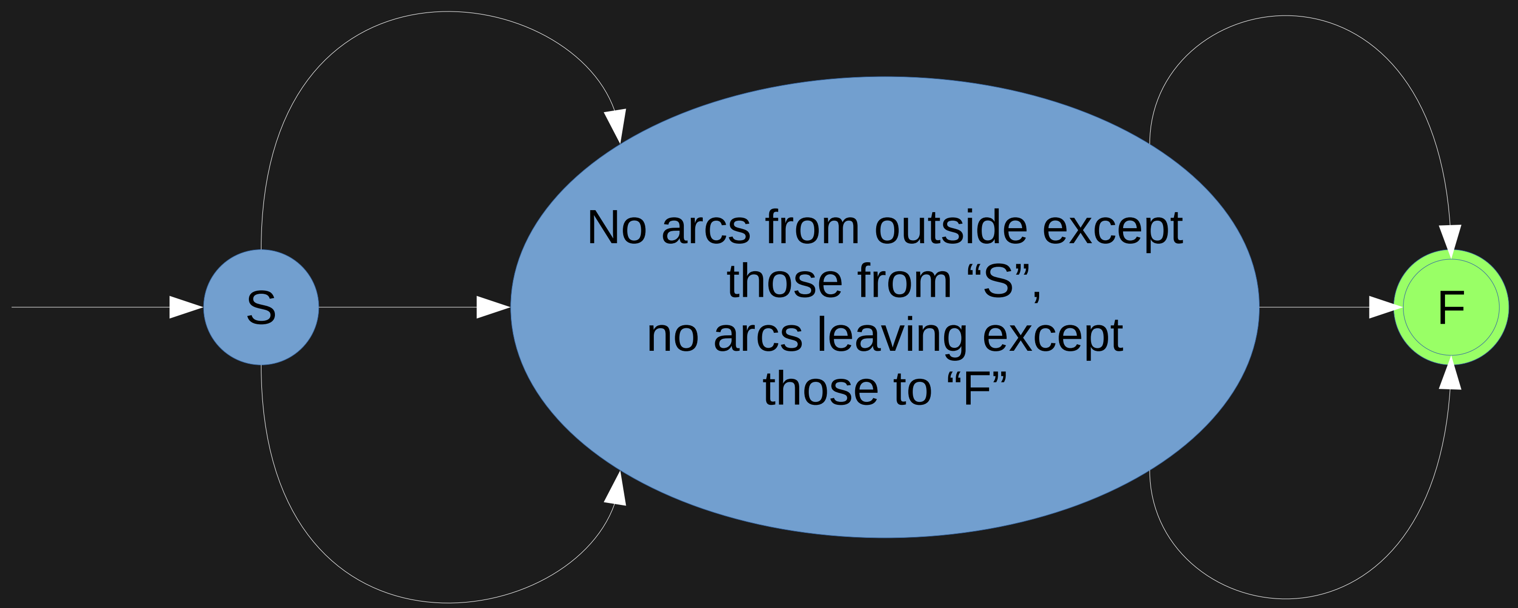


# Non-deterministic Finite Automata

- A non-deterministic finite automaton is 5-tuple: $M = \{Q, \sum, \delta, q_0, F\}$
- Q is a finite set of states
- $\sum$ is a finite set of permissible input characters
- $\delta$ is a mapping from $Q \times \sum$ to Q
- $q_0 \in Q$, the start state
- $F \subseteq Q$ is the set of final states

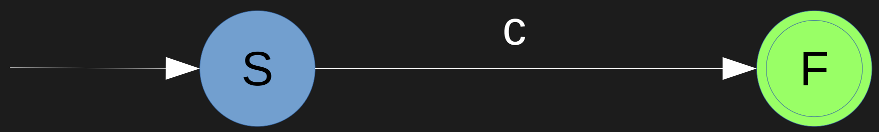
# Converting RE to NFA

- Thompson's constructions
- Only one start state, one one final state

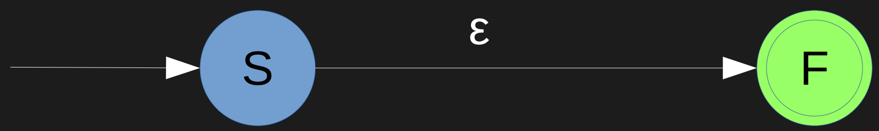


# Converting RE to NFA cont'd

- A NFA matches one character input *c*

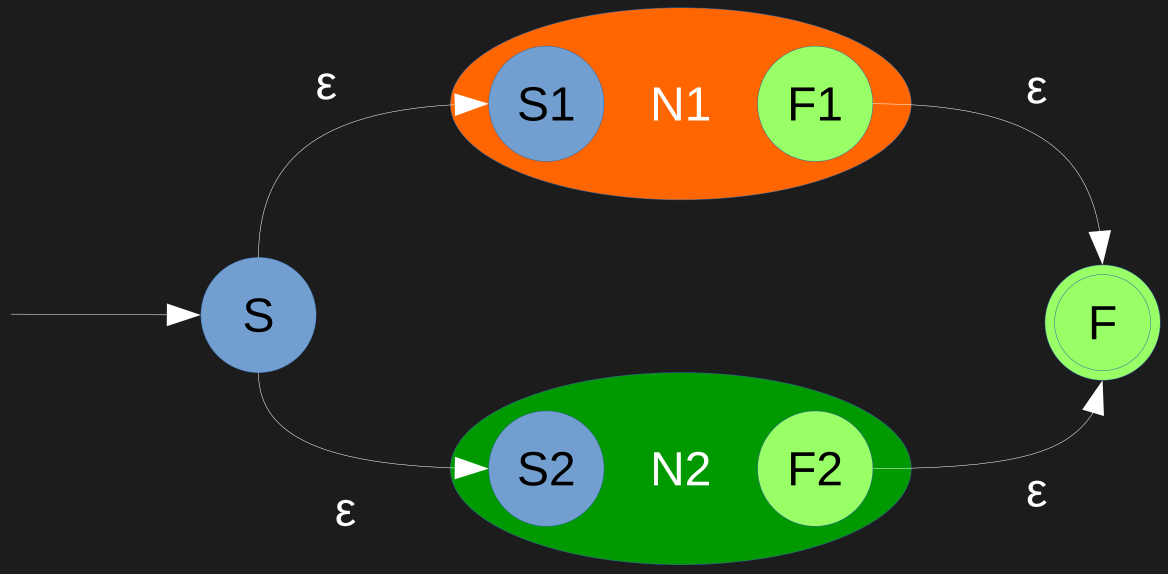



- A NFA matches an empty string: this is why NFA is non-deterministic. Because of the empty input, a NFA can in either “S” or “F” state



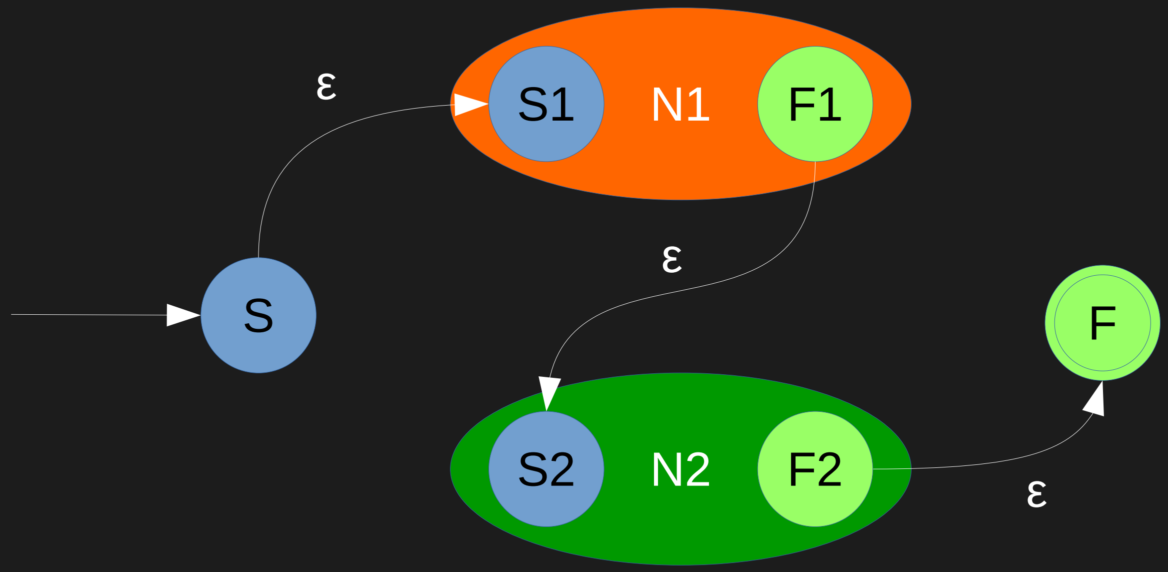
# Converting RE to NFA cont'd

- Union of two NFAs (for [ ] and '|')
  - i.e., `RE1|RE2`. Let N1 be `RE1`'s NFA, N2 be `RE2`'s NFA



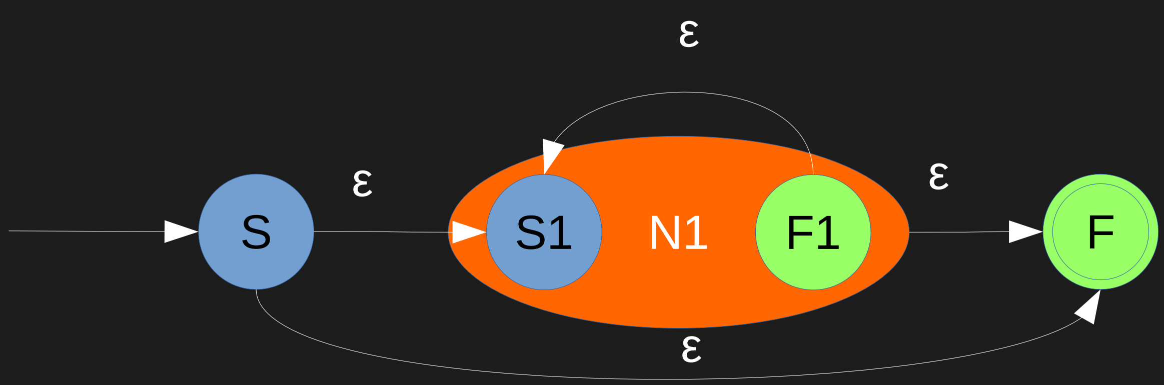
# Converting RE to NFA cont'd

- Concatenation of two NFAs
  - `i.e.,RE1RE2`. Let N1 be `RE1`'s NFA, N2 be `RE2`'s NFA



# Converting RE to NFA cont'd

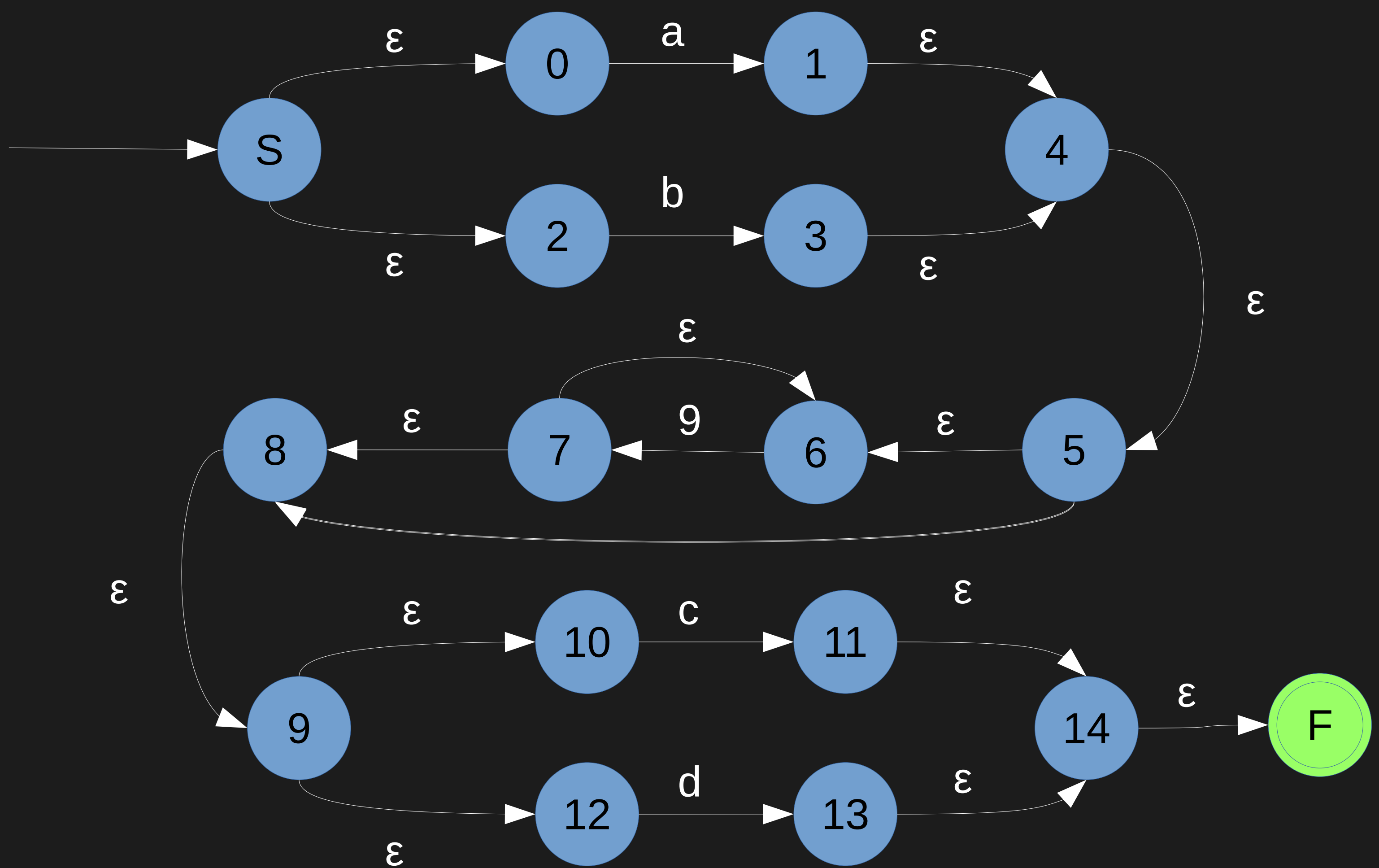
- A NFA matches `RE1*` (zero or more occurrence of pattern RE1)




- How about `RE1+`?

# A NFA Example

- Regular expression: [ab]9*[cd]



# Deterministic Finite Automata

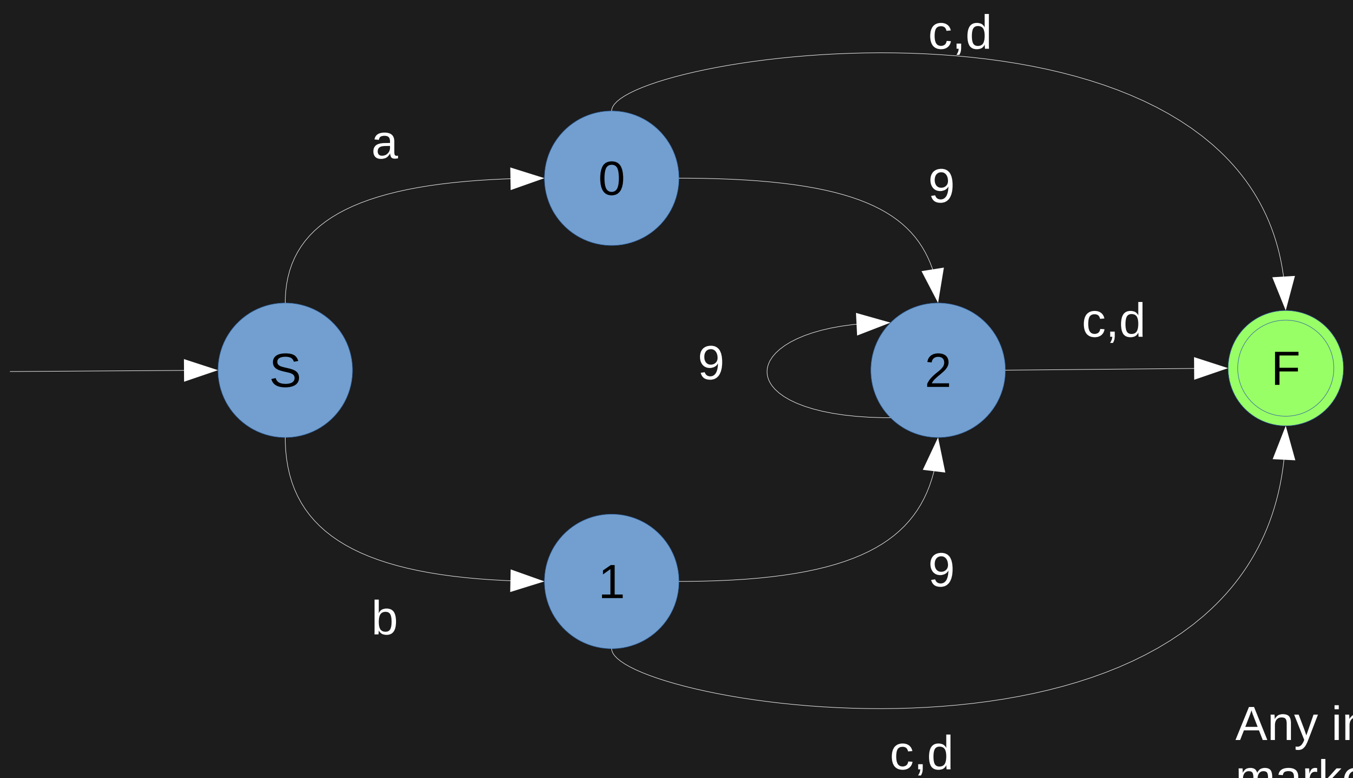
- NFA is every hard to implement, because,
  - The ε transition
  - For certain state and input there is no move
- Deterministic Finite Automata (DFA)
  - Removes the ε transition,
  - For each state and an input character, there is one and only one transition to a next state
- Every NFA can be converted into a DFA

# Deterministic Finite Automata cont'd

- A deterministic finite automaton is 5-tuple: $M = \{Q, \sum, \delta, q_0, F\}$
- The elements have similar meanings as those in NFA

# A DFA Example

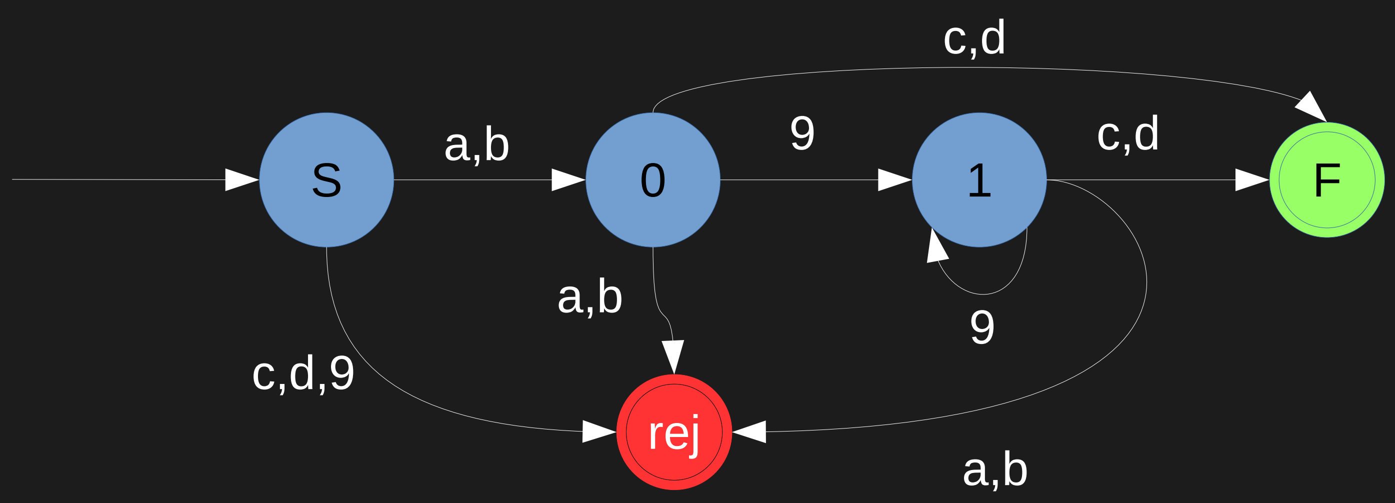
- Regular expression: [ab]9*[cd]




Any input not marked goes to rejection

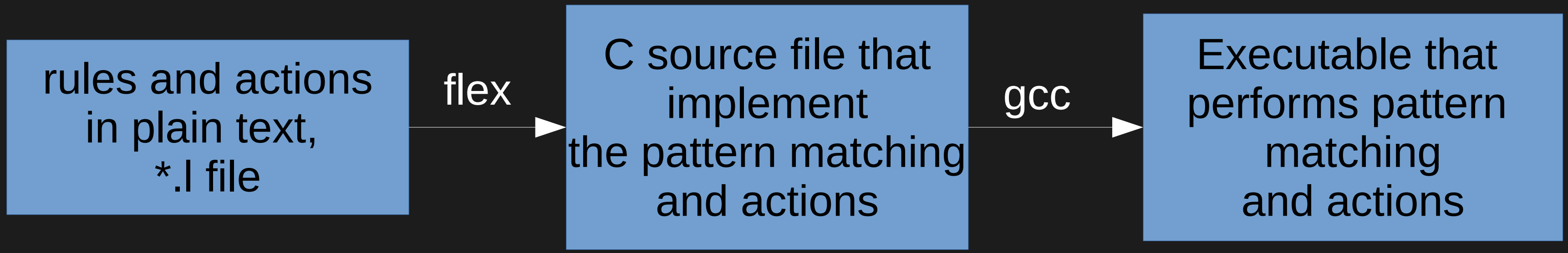
# Minimizing DFA

- There is a DFA with minimal states for any pattern
- Minimal DFA can be found by reducing a non-minimal DFA with DFA minimization algorithms
- Minimal DFA requires fewer memory to implement
- Example: [ab]9*[cd]



# Flex – A Lexical Analyzer Generator

- Given a pattern, flex automatically generate a C-program that can scan over an input string, and find the substrings that match the pattern
- Flex specification is composed of rules (patterns) and actions
  - Rules define what patterns to match
  - Actions define what to do with matched substrings



# Flex File Syntax

- Flex syntax:

```
%{
   /* Extra includes and variable declarations
      in C syntax */
%}

/* definitions for short cuts*/

%%

/* rules and actions*/
Patterns/rules      { /*actions in C */ }

%%
/* user code in C */
```

# Flex File Syntax

- Flex syntax with examples:

```
%{
   /* Extra includes and variable declarations
      in C syntax */
   #include <stdio.h>
   int global_counter = 0;
%}

/* name definitions */
DIGIT [0-9] /* declaration DIGIT to be a single number */
%%
/* rules and actions*/
/* in Flex, declared names are put in {} to use */
/* yytext is a predefined flex variable with the value of
matched substring */
{DIGIT}+      { printf("found %s\n", yytext);}

%%
/* user code in C */
int main() { yylex(); return 0;} /* yylex() starts scanning*/
```

# Flex Compilation

- Compile a flex with the following command
  - flex flex_source.l
  - A C file named lex.yy.c will be generated
- Then compile the lex.yy.c with gcc
- Example demonstration: phone number matching